BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 66 (70), Numărul 1, 2020
Secţia
MATEMATICĂ. MECANICĂ TEORETICĂ. FIZICĂ

# PHYSICAL PRINCIPLES IN REVEALING THE WORKING MECHANISMS OF BRAIN. PART II

BY

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Received: January 28, 2020
Accepted for publication: March 20, 2020

**Abstract.** The living brain is physically modelled as a universe, analogous to the existing physical model of the universe. The analogy is mathematically organized with the aid of a theory of physical quantities describing the matter from a phenomenological point of view. This theory is naturally connected with the scale of existence of a universe in general. Thus, while in the extant physical model of the universe gravitation, therefore the mass, prevails, in the brain universe the electric and magnetic charges prevail. However, the mathematical description of the two universes turns out to be the same in both cases. Therefore, physically, the charge, which is the fundamental property of the matter of living brain, should assume the same origin as the gravitational mass: it is ‘created’ under the influence of the matter. The creation goes, by and large the same way in both cases, through an algebraic combination of gravitational mass and charges, which can be appropriated as an *inertial mass*. This mass is controlled *via* the matter located at ‘infinity’ within the model, according to a general Mach principle. However, for the case of the brain universe, this infinity still needs to be defined, and we have to pursue this important task as we go along with our work. The analogy is momentarily transferred here into a mode of defining EEG and MEG signals. These are defined just like one defines the seismic signals: the Earth’s crust is thus analogous to a skull. A Yang-Mills static field is presented as an instance of electromagnetic field of such universe.

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**Keywords:** neuron; light ray; brain universe; luxon; holography; hologram; electric charge; magnetic charge; Yang-Mills fields; skull recordings.

## 1. Carrying the Physics into the Brain World

Louis de Broglie has reached the essential idea that there is such a concept as the “wave phenomenon called material point” (de Broglie, 1926). Of course, when one talks about ‘material point’ in this context, one understands the classical material point, *i.e.* a position endowed with physical attributes: mass and charge, as this concept appears in the mechanical and electrodynamical equations of motion. However, starting from frequency, de Broglie’s theory cannot be but abstract, and this is what perpetuates that strange feeling at the association wave-particle, which generated idea of probabilistic interpretation of the wave. For, the frequency is, in general, a derived concept, from the more fundamental concept, if we may, of phase, taken as a primary concept, as we have shown earlier. To wit, even today the mystery of de Broglie frequency is a subject of debate.

In the physics of brain, though, just as in the physics of heart, one cannot renounce the concept of material point, in view of the conspicuous transport of charges. Apparently, indeed, such a physics is entirely based on some ‘motions’ of charges, and the material point is usually conceived as the support of charge. Therefore, speaking of a classical material point, we still have to maintain the Newtonian point of view in constructing a theory of waves, and this can be done most naturally, and universally we should say, via the experimental *concept of phase*. Indeed, if electrodynamics allowed the introduction of the idea of synchronization as a fundamental idea of special relativity, nowadays it is time to take notice that there are *paradigms* of synchronization that connect the concept of frequency with that of phase (Acebrón *et al*., 2005), even in a possibly, well-defined statistical way (Martens *et al*., 2009). It is this type of paradigm that is usually thought for in the case of research related to living tissues. In such a case, however, we need to take into consideration the electrostatics, and even magnetostatics, rather than electrodynamics; as we said, the cases of heart and brain are well-known examples, but one can easily come up with the general idea that any physical organism on Earth runs the life within based on electric phenomena. Now, with a proper consideration based on the idea of scale invariance of the static forces in the universe, a *well-defined concept of phase* can be introduced for the general benefit of knowledge, not just for the benefit of physics *per se*. For, a concept of material point can be defined, at any space location where static forces are in equilibrium. Thus, logically, a phase, therefore a wave, should be related with such a location, making, a ‘wave phenomenon called material point’ out of it, if it is to use the phrasing of Louis de Broglie. Let us now elaborate on these ideas.

## 2. A Newtonian Characterization of Coll's Luxons

In the Newtonian stand, cosmology is indeed based on forces, and these forces are of a special type. To wit, they should still be central forces, but with a magnitude depending exclusively on the distance between bodies, and that in a quite specific way: inversely proportional with the square of that distance. Of course, in reality, such a physical structure of the universe is not possible but only in cases where the structural units of the universe – the 'bodies' of the classical Newtonian natural philosophy – are so far apart from one another, *that their dimensions are negligible with respect to the distance between them*. In discovering the Newtonian forces thus described, one can safely assume that this dimensional condition of their theoretical possibility should have already been a reality in the universe accessible to our knowledge. Indeed, otherwise we would not have the Kepler laws governing the motions of celestial bodies, which led to the invention of such forces. One can hardly say that such is the case for the brain, where the charge seems to be transmitted in a physical process, more of the nature of a propagation, than by a motion *per se*. In trying to construct a physics of brain, it helps to recall that the dimensional geometrical condition above is by no means sufficient for the task of building a cosmology. First of all, a physical structure of the very structural unit of the universe – the 'body' of the classical Newtonian cosmology – is still needed. For, in time, the mankind became aware of the existence of still *other* Newtonian forces with the very same geometrical properties. The first instance of such forces in the quotidian world of our experience was *the electric force* of Charles Coulomb. And this is how, in fact, we became aware of the necessity of a scale in characterization of the action of Newtonian forces. Indeed, a phrase like that of Hermann Weyl, justifying the modern approach of cosmology, with a universe where…

> … the electricity, which *obviously does not matter in the economy of cosmos*, we now completely dismiss… [(Weyl, 1923), §39, p.290; *our rendering and Italics*]

just reflects such an awareness.

This excerpt from Weyl, shows that we are, at least formally, allowed to think of a certain space scale as being the expression of the dominance of actions of *one or another* of the two *different forces* of the same *mathematical nature*, *i.e.* Newtonian forces. It is for these forces, that M. Berry and G. Klein proved their scale transcendence (Berry and Klein, 1984), and these are, in fact, the only forces satisfying the invariance of the dynamical description of the Kepler problem, making it a valid model of physical structures at any space scale (Mazilu, 2019). This observation entices us to take notice of what seems to be obvious by itself: the very same property of these forces that allowed Hermann Weyl to emit the speculative argument from the excerpt above, also allowed, a long time ago, to Charles Coulomb the *positive task* of experimental description of electric forces. Indeed, in the case of Coulomb, there should be just a change in emphasis: in the 'economy' *of daily life*, it is the gravitation that 'does not matter', therefore it can be dismissed, even though not quite by the same reasons the electricity at the cosmic level is dismissed. So, the apparent geometrical rigor, must be supplemented with this rigor of a physical nature, when judging the transition between universes at different space scales.

But there is more to it, and that in quite an essential way: by the presence of Newtonian forces in this outline, the *physical attributes* of the fundamental units of matter are brought to the fore, in order to regulate the transition of scales. It turns out that they allow a mathematical characterization *independent of the dimensional conditions*, mandatory for a proper mathematical description of the dynamics of Kepler problem. More specifically, the dimensional condition defining this fundamental unit of matter in the physical structure of a certain universe, is capable of accommodating the existence of physical characteristics, and this fact can be formally described by a *geometry of confinement* having, mathematically speaking, the characteristic of an *absolute* of a *Cayleyan* geometry. One has here, indeed, a concept of confinement which, physically speaking, is completely independent of any spatial condition of the kind historically involved in discovering the forces of Newtonian type. Let us see what is this all about.

First of all, in order to describe the fundamental units of matter in the same way in both cases – cosmos and daily life – the mass should be taken as the *gravitational mass*. Only in this case can one declare, as Weyl did, that for fundamental constitutive units possessing just gravitational mass and charge, in the case of 'cosmos' the Newtonian force due to gravitational mass prevails quantitatively over that due to electric charge. And only in this case, can we add that, in the case of 'daily life', the Newtonian force due to electric charge – the Coulombian force, as it is usually called – prevails over the force due to their gravitational mass, in order to be experimentally noticeable. Therefore, if we are to describe *physically* a certain universe, regardless of scale, the fundamental physical unit of this universe – the 'luxon', as we designated it every now and then, after Bartolomé Coll – should exercise two kinds of Newtonian forces acting simultaneously *in any direction* in space, *at any distance*: gravitational and electric. The universe would then be *interpreted* as an ensemble of such identical units, like the molecules of an ideal gas or those of an ideal fluid, only with Newtonian forces acting between them. The magnitude of the whole Newtonian force, acting between any two such identical fundamental physical units on a certain direction, at any distance, in a universe described in this way, can be written as

$$G\frac{m^2}{r^2}-\frac{1}{4\pi\varepsilon}\frac{e^2}{r^2} \tag{1}$$

with *r* denoting the distance between them. Here another assumption appears as necessary, and needs to be stressed once again, as an essential condition in the description of a universe: the fundamental *physical units of this universe are all identical*, having the mass $m$ and charge $e$, and the universe is described as existing in a space of gravitational constant $G$ and electric permittivity $\varepsilon$. The Eq. (1) also expresses the fact that, according to our experience, along the same direction, the two Newtonian forces act differently along one and the same direction: one is a force of *attraction*, the other is a force of *repulsion*, and the different signs of the monomials in (1) represent an algebraic writing with respect to the orientation on the direction along which the action is exerted. Thus, for instance, the universe at the cosmic scale can be characterized by

$$Gm^2 - \frac{e^2}{4\pi\varepsilon} \gg 0 \tag{2}$$

so that, using the expression of Hermann Weyl, the 'electric force can be dismissed' *at any distance and in any direction*, while the universe at the daily scale can be characterized by

$$Gm^2 - \frac{e^2}{4\pi\varepsilon} \ll 0 \tag{3}$$

so that here the 'gravitational force can be dismissed' *at any distance in any direction*, as proved by Coulomb's type experiments. Certainly the 'distance' should have different quantitative meaning in the two cases, and perhaps the 'direction' too. Whence, in our opinion, the possibility of describing *concurrent universes*, even at different space scales, to which we suggest now a positive possibility of approach, offered by the ideas of the absolute – or Cayleyan – geometry, and based on the concept of Newtonian forces.

## 3. Static Definition of Coll's Luxons

Start with the observation that the structure of a universe is always a hypothesis, so that the problem occurs: is there an ideal *static structure* of the universe, *formally the same at any scale*, that would be able to describe even the structure of the fundamental physical unit of the universe? Mathematically speaking this is always possible from all of the points of view that may be involved in the natural philosophy of such a universe. First, under the concept of interpretation by ensembles with Newtonian forces between their constituent material points, keeping in mind that the gravitational forces are always forces of attraction, while the electric forces are always forces of repulsion, the static equilibrium ensemble can exist. Secondly, this consideration is independent of the space scale of the universe, insofar as, according to Berry and Klein's theory, the action of the Newtonian forces transcends the time and space scales invariantly. Thus, for instance, a *static universe* dominated by the two Newtonian forces above, can be interpreted as an *ensemble of luxons* at any scale, provided

$$Gm^2 - \frac{e^2}{4\pi\varepsilon} = 0 \tag{4}$$

This would mean an ensemble of *identical point particles*, each one of them endowed with gravitational mass and electric charge, in static equilibrium: the forces between particles are in equilibrium in any direction for any particle of the ensemble. Such an interpretation naturally restricts the ratio between electric charge and gravitational mass of the luxons, but for now we disregard such a problem.

However there is another significant connotation of this approach of static equilibrium: as we already announced before, we can construct an *absolute geometry* based on the static equilibrium of forces, using Eq. (4) as the equation of *an absolute* in a geometry of the *physical attributes* of particles. First of all, contemplating some simplicity of the mathematics involved here, let us arrange a uniform notation based on Eq. (4), in order to simplify the algebra that follows. The terms in Eq. (4) are physically homogeneous, and both have the same composed unit (kg $\cdot$m$^3\cdot$s$^{-2}$). So, in order to make the notation uniform, we include the constants characterizing the space of residence of matter in the definition of the physical properties of the material particles, by the following transcriptions:

$$m \ \ for \ \ m\sqrt{G}, \quad e \ \ for \ \ \frac{e}{\sqrt{4\pi\varepsilon}}$$

These notations are intended to suggest that the first term in (4.4) is referring only to gravitational mass ($m$), while the second is referring only to the electric charge ($e$). This means that a universe is here interpreted by a static ensemble of identical luxons, each one of them having two physical characteristics: *mass* (gravitational) and *charge* (electric). Therefore, limiting ourselves, for the moment, to just two physical attributes: gravitational mass and electric charge, the condition to be satisfied by an ensemble of identical material points serving for the interpretation of a static universe independently of space scale should be written in the form:

$$Q(m,e) \equiv m^2 - e^2 = 0 \tag{5}$$

The left hand side of this equation thus symbolizes a homogeneous *quadratic*, *i.e.* a polynomial of second degree in its variables, whence the choice of symbol $Q$. Taken as absolute of a Cayleyan *geometry of the two-dimensional physical quantities* – gravitational mass and electric charge – of luxons, it divides the plane of these characteristics in two parts: the 'inside' part, for which we assume that the quadratic form is positive, and the 'outside' part, for which the quadratic form is negative. This is just a convention, adopted so that the 'cosmos', characterized by Eq. (2) should be *inside the absolute*, very close to its center with respect to the charge, while the 'microcosmos', described by Eq. (3), should be *outside of absolute*, very far away with respect to the gravitational mass. The true measure of these degrees of 'closeness' is, nevertheless, offered by the quantity $Q$ defined in Eq. (5). The positive part of this construction is that, once we give a metric for this geometry, we are always able to find space distributions of the quantities of gravitational mass and charge, using the *harmonic mappings*. The actual construction of these distributions may not be quite as easy as it appears, but in some cases it may be indicative of the right path of our knowledge.

For instance, the Barbilian formula for the elementary arclength of the metric geometry of this universe, is always fit for such occasions (Barbilian, 1937); see also (Mazilu *et al*., 2019) and offers, indeed, a Cayleyan metric of the plane of the *two* physical attributes, as it were, which thus we should take in the form

$$(ds)^2 = \left( \frac{mdm - ede}{m^2 - e^2} \right)^2 - \frac{(dm)^2 - (de)^2}{m^2 - e^2}$$

up to a multiplicative constant. The reason for this choice of sign for the Cayleyan metric becomes obvious by noticing that, after due calculations the metric becomes a perfect square:

$$(ds)^2 = \left( \frac{edm - mde}{m^2 - e^2} \right)^2 \tag{6}$$

and a perfect square of a real quantity should be always positive. Thus the interior of the absolute is characterized by a proper hyperbolic angle, $\psi$ say, whose variation turns out to be our metric. Indeed we have:

$$(ds)^2 = (d\psi)^2; \quad \tanh \psi \equiv \frac{e}{m}$$

The metric of physical characteristics of this universe depends on the ratio between charge and mass, a case well known in the history of physics. Only, we have to observe that here the mass is gravitational, while in the historical case the mass was inertial, as a consequence of dynamics used in describing the electron.

A problem surfaces when the charge 'splits', so to speak, *i.e*. there are Newtonian forces of *electric nature* and also Newtonian forces of *magnetic nature*. This could be the case if the Coll's luxons have *a third physical attribute*, *viz*. a *magnetic charge* and, as a result of this, a magnetic force, which is still Newtonian in character (Maxwell, 1873). As our experience shows, the magnetic poles of the same name behave exactly like the electric poles of the same charge. Therefore, a static universe interpreted by ensembles of such luxons with three physical characteristics, not just only two, will be described by a quadratic quantity reflecting the presence of both electric and magnetic charges. According to the very same theory of the Newtonian forces, this quantity must be

$$Q(m,q_E,q_M)=m^2-q_E^2-q_M^2 \tag{7}$$

instead of (4). Indeed, the vanishing of this quantity, with the electric charge $q_E$ and the magnetic charge $q_M$ appropriately defined, describes a static ensemble of luxons in equilibrium under the *three* Newtonian forces prompted by the three physical properties.

In this case, according to our metrization procedure of the physical characteristics of luxons in the matter, the same rules apply for calculating the absolute metric in the case of three physical properties of material particles. To wit, instead of (6), we have, with (7), the absolute metric

$$\left(\frac{mdq_E-q_Edm}{m^2-q_E^2-q_M^2}\right)^2+\left(\frac{mdq_M-q_Mdm}{m^2-q_E^2-q_M^2}\right)^2-\left(\frac{q_Mdq_E-q_Edq_M}{m^2-q_E^2-q_M^2}\right)^2 \tag{8}$$

In spite of its appearance, this quadratic form is a surface property, insofar as it can be expressed in two variables: $x\equiv q_E/m$ and $y\equiv q_M/m$ in the form:

$$\left(\frac{dx}{1-x^2-y^2}\right)^2+\left(\frac{dy}{1-x^2-y^2}\right)^2-\left(\frac{ydx-xdy}{1-x^2-y^2}\right)^2$$

However, this property of the expression can be seen directly through a transformation suggested by the quadratic form from Eq. (7), namely:

$$m=q\cosh\psi,\quad e=q\sinh\psi,\quad \begin{array}{l} q_E=q\sinh\psi\cos\theta \\ q_M=q\sinh\psi\sin\theta \end{array} \tag{9}$$

Inserting these into (8) results in a well-known form of the metric in $\psi$ and $\theta$:

$$(ds)^2=(d\psi)^2+\sinh^2\psi\cdot(d\theta)^2 \tag{10}$$

This is manifestly a metric of negative curvature, that can also be revealed for the relativistic velocity space, for instance. Obviously, the quantity $q$ can be calculated from (9) and amounts to

$$q^2=m^2-q_E^2-q_M^2 \tag{11}$$

but the absolute metric does not depend on it explicitly. In other words, the absolute metric, in the space of physical attributes of matter, like everywhere in fact, is a two-dimensional surface quantity, as we said. From the concept of interpretation point of view, the condition $q\neq 0$ is a nonequilibrium condition, referring to the very ensemble serving to interpretation. There are such ensembles of luxons for the interior of the absolute, as well as for the exterior of the absolute. As *per our convention*, the interior of absolute describes the matter at the cosmic scale, and in this case we have a quite well known case indicating the nature of $q$. One notices, indeed, that, towards the center of the absolute, $q$ approaches $m$, and this fact entitles us in considering it as a measure of the *inertial mass*, which in this case is not exactly equal to the gravitational mass. Rather, the Eq. (11) can be rewritten as:

$$\left(\frac{q}{m}\right)^2 = 1 - x^2 - y^2 \tag{12}$$

showing *positively* that in building the general relativity, Einstein was, indeed, entitled to 'dismiss' the difference between the gravitational and inertial mass (Einstein, 2004), because, as Hermann Weyl uttered it, we are always entitled to 'dismiss the electricity in the economy of the universe'.

## 4. A Physical Model of Charges

The Eq. (7) suggests an important identity which is, we should say, in the natural order of things we perceive in this universe. To wit, we can assume the relation

$$e^2 = q_E^2 + q_M^2 \tag{13}$$

a mathematical expression signifying the idea that we cannot physically decide how much of the perceived charge is electric and how much of it is magnetic. This equation represents what we believe as one of the most striking pages of the modern natural philosophy. And unique of its kind too! It therefore deserves a little more elaboration, inasmuch as this turns out to be quite useful in developing a theory of the universes based not on mechanical inertia, but static universes, based on the electric and magnetic 'inertia', as it were. These universes can model life sustaining organs, like the brain for instance, which is our subject-matter in this work. Only, we have to pay close attention to an important issue in such a modeling: the living organs do not exercise their function by themselves, but in connection with other organs.

The essential physical trait of a universe, as physically conceived nowadays, is its *uniqueness*. This means that the universe is conceived as the world we live in, and, no matter of their space scale, all things material in this universe are to be described as simultaneously existing in such a world, and having the same fundamental physical structure. For instance all of them have the same property of inertia, all of them have to be described by the rules of dynamics *etc*.: there is no discrimination in dominance of the physical attributes, of the kind we have shown above. This philosophy has always promoted the synthetic approach of the whole, usually represented as a set of parts put together by external connections, an approach that may serve the social life to a certain extent, but surely does not touch the fundamental laws of existence. The reason is that the connections in question are, as a rule, only those controllable by human means, and these are quite limited in number, while a part, in its capacity as organ, is itself a universe, with an unlimited number of connections, as it were.

Perhaps these ideas are not quite so clear for everybody, but the previous manner of constructing the physical universe, surely can display the essential points as clearly as possible. First, the matter is there present by a density related to its three attributes – the mass and the two charges – concurrently. The *usual* approach in physics is a 'one-by-one', as it were: the mechanics deals in the density of mass, the electrostatics deals in the density of charge, and so on. But this is not all of it: in the construction of a model universe, we assume that the *mass is dominant at any scale*, be it microcosm, daily life of macrocosm, and in the very same way, mathematically expressed by the positive sign of the expression (11). The Einstein's conclusion, sanctioning the results of Eötvös experiments, according to which the inertial mass can be safely considered as identical to gravitational mass, is always presented as independent of space scale where the universe is contemplated. In other words, the *universe is unique*, as we said before, and the physics of its parts is independent of scale. With the model universe presented by an absolute geometry of the physical attributes, the things go in an essentially different direction of thinking.

To wit, that model was constructed based on the explicit *quantitative dominance* of the gravitational mass in each and every one of its identical representative constituents, the luxons. Thus, the whole metric geometry of the universe is presented based on an inequality:

$$q^2 \equiv m^2 - q_E^2 - q_M^2 > 0 \tag{14}$$

assumed to be valid in this universe for every material particle of it. We called the quantity *q inertial mass*, and if we think of it in classical terms, this universe is filled with matter whose constitutive formations all satisfy (14) in a way or another. Dealing in inertial mass, so to speak, the universe 'extends' until the condition $q^2 = 0$ becomes effective, which represents the infinity in its definition: *the absolute*. Physically, the absolute represents a world of particles having null inertial mass, just like de Broglie's photons in the case of light, or the Coll's luxons here. But here the analogy stops: these null mass particles – which we call luxons – are, unlike the old photons, *static fictitious particles*, serving merely for the wave-mechanical *interpretation* of matter.

Now, this is only what the mathematics say. Physically, the inequality (14) is taken as characterizing a universe in its entirety, which becomes unique in this way. The inertial mass is thereby controlled by the matter located beyond the spatial possibilities of human accessibility, and the equality between inertial and gravitational mass should mean that such matter prevails quantitatively regardless of scale. Therefore, regardless of how far can we extend the limit of our observations in space, the matter outside that limit is quantitatively dominant. In other words, we cannot access but an infinitesimal amount of matter in any finite space, and regardless of how much we extend our capability of knowing the universe, there will always be 'missing mass', if it is to use the guise of modern theoretical physics. Not so if we properly use the geometrical model in its fullness!

Indeed, the previous physical assumption (14) leaves aside the condition

$$q^2 \equiv m^2 - q_E^2 - q_M^2 < 0 \tag{15}$$

as being physically intractable. However, from the point of view of an absolute geometry based on (14), this condition represents points *outside* the absolute, and these are describable by an *angular metric*, as it were, inasmuch as the root of this quantity is purely imaginary. In other words, these points are prone to represent *phases*, like the de Broglie phases, which can be described as such with respect to the speed of light. They characterize, indeed, the argument of the waves associated with particles in the usual physical cosmology, and these waves propagate with a speed higher than that of light. However, from a general physical point of view, we can have here a world where the charge prevails over the gravitational mass, and this world is certainly real: it can be either the world of our daily life or, even better, at another scale, the microcosmos of particles constituting the matter. These satisfy to the inequalities:

$$q^2 \equiv q_E^2 + q_M^2 - m^2 > 0 \tag{16}$$

Incidentally, one can thus have a logical explanation why the wave mechanics made its mark especially in the microcosmos. The general idea is that, mathematically and physically, a universe proper cannot exist without its two worlds – of particles and waves – at different scales, representing the structure of matter contained in it. Taken as such, the universes can be multiple with no problems: they can be described by the position of their absolutes with respect to each other. However, inasmuch as this description can become awfully involved, for the moment we shall describe a *unique universe* of the microscopic world based on the quantitative inequality (16), just like we did before for (14). This represents a world where *the charge prevails over the inertial mass*. This would be, unconditionally, the world of human brain, or of human heart, but microscopical in character, so that we can apply all the previously developed wave-mechanical theory.

## 5. A Way of Charge Transmission: Solitons

In order to elucidate what that 'microscopical in character' means, we need the metric of this cosmology: if it is to extend the analogy to details, the analog of Eq. (10) is the best place to start, in fact the only place to start. The metric still has the geometrical expression (8) up to its sign, but with mass and charges from Eq. (9) given by a transformation satisfying naturally the defining condition (16) for this universe:

$$m = q\sinh\psi, \quad e = q\cosh\psi, \quad \begin{matrix} q_E = q\cosh\psi\cos\theta \\ q_M = q\cosh\psi\sin\theta \end{matrix} \tag{17}$$

Thus, instead of Eq. (10), we have now for the metric of this universe

$$(ds)^2 = (d\psi)^2 - \cosh^2\psi\cdot(d\theta)^2 \tag{18}$$

but the analogy between this universe and the customary one seems to stop here, short of any conclusion. This is due to the fact that we do not know what to do with this metric: we do not have as our disposal the Einsteinian guiding principles of general relativity, which places its stakes on the metric, in order to describe the motion. However, as we already have noticed before, the motion in the brain universe can only be a metaphor, at the most: the charge is 'transmitted' in the brain, it is not 'moving' in the usual sense we assign to this word. This impasse should, nevertheless, not be taken seriously.

Fact is that the description of motion is not the only lesson we have to take from the general relativity, especially when it is to carry this lesson over to a cosmology. In view of the meaning of the parameters $\psi$ and $\theta$, as this meaning comes out of the Eqs. (9) or (17), we may be interested in knowing how these parameters behave in the host space of the matter, rather than with respect to a motion. Indeed, the motion is referring to a material point, while the distribution of mass and charge can be a collective property of ensembles of such material points. According to the precepts of general relativity and wave mechanics the distribution of charge and mass in the host space would then involve the concept of field. And, in this respect, we have to recall that Frederick Ernst has shown that knowing the properties of the host space of matter, the Einstein's field equations are reducible to a variational principle (Ernst, 1968; Ernst, 1971), namely a Dirichlet-type principle defining some so-called harmonic applications - see (Misner, 1978), for an account of using the harmonic applications in physics. The general Dirichlet's principle can be presented here as follows (Eells and Sampson, 1964).

An application of the physical attributes onto the host space should have, in general, two components, if it is to consider the metrics (10) and (18) of the manifolds of these physical attributes. Only in the case of charge and gravitational mass only, the dimension of the manifold is one (the variable $\psi$). Such an application can be represented therefore as having two components, functions of position in the host space. In a uniform notation, we put:

$$y^\alpha(\mathbf{x}) = f^\alpha(x^1, x^2, x^3), \quad y^1 \equiv \psi, \qquad y^2 \equiv \theta \tag{19}$$

If the host space is characterized by the tensor ***h***, and the metric tensor of the manifold of physical attributes – let us call it a surface for now, in order to highlight its usual dimension – is ***g*** say, then by application (19) the matter induces a deformation in space, described by the following variation of the its metric tensor:

$$\Delta\mathbf{h} \equiv \mathbf{f}^{*}\mathbf{g};\quad \Delta h_{ij} = f_i^{\alpha} f_j^{\beta} g_{\alpha\beta}\quad f_i^{\alpha} \equiv \partial y^{\alpha}/\partial x^{i} = (\nabla\mathbf{f})_i^{\alpha} \tag{20}$$

Thus, we can define a one-parameter family of metric tensors 'updated by matter', as it were:

$$\mathbf{h}_{\lambda} = \mathbf{h} + \lambda\cdot\Delta\mathbf{h} \tag{21}$$

where $\lambda$ is the parameter of this family. The volume of space filled with matter, say $H$, to be in some accordance with the notation for the metric tensor, can be calculated with the benefit of the deformed metric, the way such a volume is usually calculated, *i.e*. by an integral:

$$\begin{aligned} V_{\lambda}(H) &\equiv \iiint_{H} \sqrt{\det(\mathbf{h}_{\lambda})}(dx^1\wedge dx^2\wedge dx^3) \\ &= \iiint_{H} \sqrt{\det(\mathbf{h}+\lambda\cdot\Delta\mathbf{h})}(dx^1\wedge dx^2\wedge dx^3) \end{aligned} \tag{22}$$

Then, with this functional, we can define the *energy of application* (19) as:

$$\begin{aligned} E(\mathbf{f}) &\equiv \left.\frac{d}{d\lambda}\right|_{\lambda=0} V_{\lambda}(H) \\ &= (1/2)\iiint_{H} [tr(\mathbf{h}^{-1}\Delta\mathbf{h})]\sqrt{h}(dx^1\wedge dx^2\wedge dx^3) \end{aligned} \tag{23}$$

where $h$ is the determinant of the extant host space metric ***h***. The scalar integrand:

$$e(\mathbf{f}) \equiv (1/2)[tr(\mathbf{h}^{-1}\Delta\mathbf{h})] = (1/2)h^{ij} f_i^{\alpha} f_j^{\beta} g_{\alpha\beta} \tag{24}$$

counts as a density of energy of the application ***f***(***x***). Considering the functional $E$(***f***) stationary, a regular variational principle produces the system of partial differential equations

$$h^{-1/2}\partial_i(\sqrt{h}h^{ij} f_j^{\alpha}) + \Gamma_{\beta\gamma}^{\alpha}(\mathbf{g}) f_i^{\beta} f_j^{\gamma} h^{ij} = 0 \tag{25}$$

where we used some traditional notations for the partial derivatives and the Christoffel's symbols of second kind of the surface metrics. Ernst discovery is that the Einstein gravitational equations for the vacuum and electrovacuum fields can be reduced to such an equation for a variant of the metric (10) involving a complex potential (Ernst, 1968).

Thus the analogy between the customary universe of physics and the brain universe does not really stop: it just needs to be properly conducted. For our convenience, we illustrate now the Ernst theory in the brain universe, therefore with reference to the metric (18), assuming that the host space is Euclidean. The principle of harmonic applications will produce the system (25) in the form

$$\nabla^2\psi + \sinh\psi\cosh\psi\cdot(\nabla\theta)^2 = 0;\quad \nabla[\cosh^2\psi\cdot(\nabla\theta)] = 0 \tag{26}$$

which shall be used to describe the location of charge in space and the characteristics of its distribution. One can see that in the case of only electric charge and mass, the only equation remaining here is the Laplace equation for the ratio between charge and mass - see Eq. (6). However, we are really interested in a general manifold of physical attributes, having therefore two dimensions.

Indeed, speaking of the charge at least, is known that it always realizes stationary distributions on surfaces. Again, a manifold coordinated by the parameters $\psi$ and $\theta$ as functions of position in space can also be organized as a surface in space. So, we can try to solve the system (26) with respect to a parameter representing the distance from a plane in space, that, incidentally, can be the tangent plane of such a surface. That dependence can be realized via a linear form in the position coordinates, *i.e.* through a dot product of the form $\xi \equiv \boldsymbol{a}\cdot\boldsymbol{x}$. In this case, the system (26) can provide solitonic-type solution: distribution described by the parameters $\psi$ and $\theta$ as functions of the distance of a plane in space. Eq. (26) can be written as a system of two second order ordinary differential equations:

$$\psi'' + \sinh\psi\cosh\psi(\theta')^2 = 0;\quad [\cosh^2\psi(\theta')]' = 0 \tag{27}$$

with a prime denoting the derivative with respect to $\xi$. The second of these equations can be integrated right away, and gives the result

$$\theta' = \frac{C}{\cosh^2\psi} \tag{28}$$

where $C$ is a constant of integration, which we take as real number. Using (28) in the first of equation from (27) gives

$$\psi'' + C^2\frac{\sinh\psi}{\cosh^3\psi} = 0$$

This can again be integrated directly, multiplying it by $2\psi'$, which leads to a first implicit integral

$$\psi'' + C^2\frac{\sinh\psi}{\cosh^3\psi} = 0 \quad \therefore \quad (\psi')^2 = C_1^2 + \frac{C^2}{\cosh^2\psi}$$

with $C_1$ a new real constant of integration. There is no sign problem in the right-hand side here: all things considered real, the expression is positive, and the square root does not involve any complex numbers. After some calculations we can arrange here a total differential

$$\frac{d(\sinh\psi)}{\sqrt{\sinh^2\psi + \cosh^2\psi_0}} = C_0(d\xi)$$

and therefore, a direct integration to a closed form solution:

$$\sinh\psi = \cosh\psi_0 \cdot \sinh[C_0(\xi - \xi_0)] \tag{29}$$

where $C_0$, $\psi_0$ and $\xi_0$ are real constants of integration. Using this last result in Eq. (28) we get the exact differential

$$d\theta = C_0^2 \tanh^2\psi_0 \frac{d\xi}{\sinh^2[C_0(\xi - \xi_0)]}$$

that can be integrated right away giving the closed form result:

$$\frac{\theta_0 - \theta}{C_0 \tanh^2\psi_0} = \frac{\cosh[C_0(\xi - \xi_0)]}{\sinh[C_0(\xi - \xi_0)]} \tag{30}$$

$\theta_0$ being still another integration constant. In getting these results one can use tabulated formulas in order to avoid too much calculational effort. The Eqs. (29) and (30) provide our solution to differential system (27). It depends on four real parameters: $\theta_0$, $\xi_0$, $\psi_0$ and $C_0$, which can be fixed by some boundary conditions, appropriate to the problem. This may turn out to be a routine, but it does not touch the essentials of the physical argument. However, that argument is strongly influenced by the fact that the vector ***a*** is completely arbitrary in the definition of the parameter $\xi$.

While, in order to extract any proper physical meaning of the results right above, we need to shift the reason to a proper phenomenology of the brain universe, we feel like giving the reader an idea about the finality of these results right away. This finality is independent on any physical explanation – on the contrary, such an explanation rests upon its understanding – and regards the idea of de Broglie ray, which is a concept we should contemplate here as part and parcel of the physics of brain - see (Mazilu *et al.*, 2019). Indeed, in obtaining the regular classical meaning of the phase – equation, Louis de Broglie used a condition on the amplitude of the wave associated with the particles: the ratio of that amplitude to its derivative along the ray vanishes at the position of particle. This has as consequence the fact that as one approaches the moving particle at constant time along the ray, the amplitude varies inversely with the distance to particle. This condition is the most general condition of definition of an ensemble of 'contemporary' particles necessary to a process of interpretation, particularly an ensemble of simultaneous positions. However, there is here a subtle reference to the fact that the definition depends on the physical characteristics of its element. If this element is participating in different waves, propagating in different directions in space, the de Broglie's condition is directional: we need to know with respect to which direction that 'inverse proportionality' is reckoned. This can only be judged with respect to the tangent plane of the wave touching the point at a certain moment of time, and that distance is measured by the parameter $\xi$ introduced by us right above.

## 6. Switching to a Phenomenology of the Brain World

In constructing the physics of brain, we have to overcome one of the toughest issues that the idea of analogy between universes can encounter: there is no motion in the brain, at least not in the classical sense of this word, or even in the sense of relativity for that matter. This is why we have to rely here on the idea of wave in the first place. Solving this issue would require us to move the analogy over to another plane of thinking, in order to get read of the classical understanding of inertia, and to replace it with a concept appropriate to the brain world. This chapter of our work is dedicated to this subject. The results will be used to physically describe the 'brain waves' so to speak.

## 7. The Natural Philosophy of Charges

In spite of the fact that we have to leave it aside the usual concept of inertia in the brain world, the physical basis of working of this world can still be understood by a sheer analogy with the extant 'inertial universe' of physics, as it were. This statement has, nevertheless, a precise meaning here: to wit, the charge $q$, in a charge-dominated universe, just like the inertial mass in the regular Newtonian universe, should be *induced by the matter spatially located outside our perception limits*. Only, as there are two kinds of charges, things become a little more involved than in the case of inertial mass: the charge can be *randomly induced* as electric, as well as magnetic charge. Due to the Eq. (13) a phase proper is involved in this description, therefore a de Broglie-type wave can be associated with the constitutive particles of this universe.

Now, if we assume the identity (13), the charge $e$ splits, 'Euclidean-wise' so to speak, into an electric charge $q_E$ and a magnetic charge $q_M$. In today's theoretical physics this speaks out of a specific invariance of electromagnetic theory: the invariance with respect to what is today generally known as the *duality rotation* in electromagnetics. Expressed simply, this rotation is just a Euclidean rotation that leaves invariant the *experimental* electric charge $e$ of the 'split'suggested in Eq. (9):

$$q_E = e\cos\theta, \quad q_M = e\sin\theta \tag{31}$$

Here $\theta$ is the angle variable describing the split among the possible experiments with the charge. Some of these experiments involve the 'non-gravitational force' as it were, in its 'electric instance', some in its 'magnetic instance', and that assignment is unrecognizable in experiment, therefore uncontrollable. The argument then goes on to declare that the Maxwellian theory, with its privileging the electric charge monopoles, and eliminating the magnetic charge ones, represents just one possible choice of the split angle – more to the point, $\theta = 0$ – among infinitely many others. The magnetic pole case would be represented here by the choice $\theta = \pi/2$ for the experiments involving charges. Theoretically, the extra degree of freedom, represented in concrete terms by the existence of duality rotation with respect to which the whole charge behaves invariantly, can be allocated to the known possibility of transition between *field description* by electric and magnetic intensities, and *Maxwell stresses* description (Katz, 1965). As to the *natural philosophical* reason of this possibility, it is indeed quite remarkable. Quoting:

> It is frequently pointed out that the crucial difference between electric and magnetic phenomena, which underlies this dissimilarity (*between Maxwell equation for the electric field and Maxwell equation for the magnetic field, n/a*), is that electric charges *occur in nature* as monopoles whereas magnetic charges do not so occur, *but only as dipoles*, and higher poles. This is demonstrated, for example, by breaking a permanent magnet in two. In so doing one *does not obtain free north and south poles*: each piece has again both polarities of equal magnitude. The mathematical formulation of this situation leads then to the equation div$\mathbf{B} = 0$. On the other hand, electric charges can be obtained free, it is said.
>
> This reasoning is incomplete and deceptive. It is true that a permanent magnet *has equal and opposite magnetic charges near its ends*, and that by breaking the magnet in two and separating the parts and *inserting a chunk of empty space between them* new poles will appear on the new surfaces. But it is equally true that *electric charges occur only in equal pairs of opposite sign at opposite ends of a chunk of vacuum*, for example, by rubbing a rubber rod with a catskin and then separating the two. The vacuum between the rod and the catskin is analogous to the permanent magnet in that it has charges of equal magnitude and opposite sign at opposite ends. If we now *break the vacuum space between the two ends in two*, by *inserting, for example, an isolated conductor between them*, then charges are induced on the metal-vacuum interfaces such that each of the two chunks of vacuum carries again zero total charges at its ends. A well-known variation of this procedure is the so called ice-pail experiment of Faraday. One can pursue this reasoning further. The conclusion is that also electric charges occur only in pairs which can be looked at as

> the result of polarization. The only difference is that *magnetic poles appear as a result of polarization of a region of space filled with matter* (and so far no region of space filled with vacuum has yielded to polarization of this kind), whereas electric charges appear as *a result of polarization of a region of space filled with vacuum as well as at one filled with matter*.
>
> Logically and formally it is therefore possible to treat electricity and magnetism completely similarly, as long as *one is willing to treat a region of space filled with vacuum on the same footing as a region of space filled with matter* (Katz, 1965).

This last sentence is, in our opinion, of considerable importance for a natural philosophy of the brain world: rarely, if ever, is one willing to recognize in physics that while theoretically treating the vacuum *unreservedly* as a material, one has also the obligation to *think of it* as of a material of the daily life, as of a 'chunk' in the expression of Katz. It seems to us that the electromagnetic theory has more to show than it appears at the first sight. For instance, not only it enforced the relativity at the time it did, an obligation that turned the physics upside down, as it were, but also tells us how to turn our very intuition into concept, and that in a right way. As far as we are concerned, the message of the previous excerpt is quite clear: the existence or non-existence of the singular magnetic poles – the magnetic monopoles of today's physics – is pending on the necessity of *describing the electromagnetic field by Maxwell stresses*. This requires, indeed, more than the wave-mechanical idea of interpretation – it calls for a reverse interpretation –to which we shall return later on, on a special occasion of the physics of brain.

## 8. Three-Dimensional Space of Charges

The Eq. (13) unlocks a circumstance in physics similar to that unlocked by Fresnel's mathematical treatment of diffraction. Just like the light in that historical case, here too, but this time *the charge*, may be treated via harmonic oscillators. And, when recalling the historical fact that light can be described as an electromagnetic phenomenon, we are inclined to believe that this kind of treatment for the charge prevails ontologically over that of Fresnel for light. Fact is though, that, just like in the case of light, a *dynamical* problem based on the equation of motion of harmonic oscillator does not make sense for the case of charge, the very same way it made sense for the light in Fresnel's take. Specifically, like in the old case of the Fresnel's theory of light, the second order differential equation jumps our mathematical reasoning in the case of charge too, by the very nature of the mathematical apparatus. Indeed, if we assume (13), we should further assume that

$$e^2 = q_E^2 + q_M^2 \quad and \quad \begin{aligned} q_E^2 &= q_1^2 + p_1^2 \\ q_M^2 &= q_2^2 + p_2^2 \end{aligned} \tag{32}$$

Then, as the Eq. (31) suggests, the geometry of charge space is *a priori* dictated by a second order differential equation having the phase $\theta$ as independent variable replacing the classical time, but this variable actually represents the split of charge in different experiments involving the electricity. The Eq. (32) extends the logic that led to the Eqs. (9) or (17). It says that what happens in the case of electric charge can just as well happen in the case of magnetic charge: when physically perceiving the magnetic charge, there is an uncontrollable electric part of it, and an uncontrollable magnetic part of it.

Now, as we said, just like in the case of Fresnel's light, the second order differential equation, valid here by the geometrical nature of Eq. (32), may be physically endorsed by the second principle of mechanics, inasmuch as a second order differential with respect to a phase that may be taken as time, can always be appropriated as an acceleration. And, just in the classical case of light, the procedure is doomed to a dynamical nonsense, as the inertial mass effects are negligible, perhaps nonexistent. However, for the benefit of future developments here, we need to reveal that the split of charge is equivalent to an extremum property of the Eq. (13), that directs the physical reasoning away from dynamics, just as it happened in the classical situation, where the reasoning was directed to electrodynamics in the case of light. Indeed, take the case of one of the quadratic equations from among those of (32), to wit, that from Eq. (4.13). Assume an experiment that reveals the value $e$ for the charge. Then, naturally, we need to know how much of this charge is assigned to electric property and how much is assigned to magnetic property. We need a constraint in order to do that and, historically speaking at least, physics has assigned a linear connection between the two charges. This represents the natural condition that a real charge can always be represented as located on a straight line in the charge plane. So, if we denote the alleged experimental charges by $q_1$ and $q_2$, we are entitled to find the extremum of the function of two variables

$$e^2 = q_1^2 + q_2^2 \tag{33}$$

provided the linear expression, conveniently defined by the electric and magnetic properties as a bilinear form, suggested by the differentials in the metric (8):

$$q_2 q_E - q_1 q_M \tag{34}$$

has a given value. The extremum of charge is then assured for

$$q_1 = -e\cos\Phi, \quad q_2 = e\sin\Phi, \quad \tan\Phi \equiv q_M / q_E \tag{35}$$

In other words, the *split angle* from Eq. (31) is the supplement of phase $\Phi$ due to the indecision of experimental charge. This means that we have a theoretical reason for considering the idea of charge plane, pending on the physical meaning of the linear connection (34) between the two kinds of charge. As it turns out, there is a reason, not only for that, but also for the very quadratic forms from the Eq. (32).

In (Mazilu *et al*., 2019), we have presented an analog of the coordinate along the physical ray, necessary, according to Bartolomé Coll, for the construction of a coordinate system. The ray surface is always transversal to this coordinate line, and we have to describe this situation *in terms of the coordinate line alone*. Assume that it is represented as a vector in a certain reference frame, that vary only across the ray, more to the point perpendicularly on the coordinate line. In the case of gravitational field of the Earth – which, more generally, is also the case of a Schwarzschild field in general relativity – this represents the motion of charges lifted from the crust on atmospheric vortices, in order to create the electricity in the atmosphere. One needs to describe this situation exclusively with respect to the coordinate line as a *primary concept*: any external inclusion in the description would involve the coordinate system to which the coordinate line belongs, and therefore the coordinate line would not be a primary concept anymore.

Denote $\boldsymbol{m}$ the vector long the coordinate line represented by a certain physical ray. In their capacity as coordinates along the ray, the components of the vector $\boldsymbol{m}$ reveal the *transversal* velocities:

$$\dot{m}_1 = \nu(m_2 - m_3), \quad \dot{m}_2 = \nu(m_3 - m_1), \quad \dot{m}_3 = \nu(m_1 - m_2) \tag{36}$$

where $\nu$ is a constant frequency and an overdot means time derivative as usual. These equations represent a motion perpendicular to the ray, as in the case of light. Now, the equations of this motion can be 'decoupled', so to speak, by successive differentiation on time. The conclusion is that each component of the motion described by (36) is solution of the very same third order differential equation, *i.e*. the vector $\boldsymbol{m}$ is solution of the same equation:

$$\dddot{\boldsymbol{m}} + 3\nu^2\dot{\boldsymbol{m}} = \boldsymbol{0} \tag{37}$$

We are searching for constant integrals of this motion, starting from the differential system (36) and trying to find exact differentials that could offer us a physical interpretation to the parameter of continuity. The most obvious method involves linear forms in coordinates. For instance, we can derive the exact differential equivalent to the system (36):

$$\frac{adm_1 + bdm_2 + cdm_3}{(c-b)m_1 + (a-c)m_2 + (b-a)m_3} = \nu dt \tag{38}$$

with constant ($a$, $b$, $c$). Such exact differentials can exist under the conditions

$$c - b = \lambda a, \quad a - c = \lambda b, \quad b - a = \lambda c \tag{39}$$

with $\lambda$ a parameter. This means that the left-hand side of the Eq. (38) is an exact differential only for the cases in which $\lambda$ has as values the roots of the cubic equation:

$$\lambda(\lambda^2+3)=0$$

representing the condition of compatibility of the system (39). This is also the characteristic equation for the differential Eq. (37). In terms of the three roots of this equation, to wit, 0 and $\pm i\sqrt{3}$, the following three complex integrals can be constructed with respect to some initial conditions at the time $t = 0$:

$$\begin{aligned} m_1+m_2+m_3 &= m_1^0+m_2^0+m_3^0 \\ m_1+j^2m_2+jm_3 &= e^{i\nu t\sqrt{3}}(m_1^0+j^2m_2^0+jm_3^0) \\ m_1+jm_2+j^2m_3 &= e^{-i\nu t\sqrt{3}}(m_1^0+jm_2^0+j^2m_3^0) \end{aligned} \tag{40}$$

Here $j$ is the *cubic root* of unity, as a counterpart of $i$, which is the *square root* of negative unityin the case of usual complex numbers. The three complex variables from the left hand side of this equation are related, and in quite a few ways at that, to the name of Paul Appell, and they have a tremendous importance of principle, both from physical (Appell, 1893) as well as from mathematical point of view (Appell, 1877).

Start by noticing that the first of the integrals (40) is a constant of motion. Another constant of motion is quadratic, and can be obtained from the product of the last two of them, *i.e.*:

$$(m_2-m_3)^2+(m_3-m_1)^2+(m_1-m_2)^2 = const \tag{41}$$

The trajectory is then to be found in the intersection of this quadric with the real plane given by the first equality from Eq. (40), therefore it also belongs to the quadric

$$m_2m_3+m_3m_1+m_1m_2 = const \tag{42}$$

and thus on a sphere.

Indeed, according to Eq. (37), the charge can be itself represented as a genuine periodical process having as components the solutions of the differential equation

$$\ddot{\boldsymbol{m}}+3\nu^2\boldsymbol{m}=\boldsymbol{c} \tag{43}$$

where $\boldsymbol{c}$ is an arbitrary constant vector introduced by three constants of integration. Such a phase space obviously generalizes, by dimension at least, the phase plane of a regular harmonic oscillator: as one can easily see, if we settle for a plane of coordinates in (36), we get a two-dimensional harmonic oscillator. The solution of (43), on the other hand, is offered by the velocity vector:

$$3\nu^2\boldsymbol{m}=\boldsymbol{c}+\boldsymbol{a}\cos(\nu t\sqrt{3})+\boldsymbol{b}\sin(\nu t\sqrt{3})$$

with $\boldsymbol{a}$ and $\boldsymbol{b}$ some initial conditions. It is located on the homogeneous quadratic cone, having the equation

$$[(\boldsymbol{b}\times\boldsymbol{c})\cdot\boldsymbol{m}]^2+[(\boldsymbol{c}\times\boldsymbol{a})\cdot\boldsymbol{m}]^2-[(\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{m}]^2=0 \tag{44}$$

The coefficients are here decided by the initial conditions.

Let us continue along this line for a little while. From the system (40) we can get the one-parameter group equations of the very process thus described by Eqs. (36). Namely, after taking the inverse of the appropriate matrix in (40) we end up with

$$\boldsymbol{m}=[\boldsymbol{1}+(2/3)\boldsymbol{F}(t)]\cdot\boldsymbol{m}_0 \tag{45}$$

where $\boldsymbol{1}$ is the 3×3 identity matrix, and $\boldsymbol{F}(t)$ is the matrix:

$$\boldsymbol{F}(t)=\begin{pmatrix}\cos\phi & \cos(\phi-2\pi/3) & \cos(\phi+2\pi/3)\\ \cos(\phi+2\pi/3) & \cos\phi & \cos(\phi-2\pi/3)\\ \cos(\phi-2\pi/3) & \cos(\phi+2\pi/3) & \cos\phi\end{pmatrix}$$

with $\phi(t)\equiv t\cdot(v\sqrt{3})$. This matrix is singular: its determinant is

$$\det\boldsymbol{F}(t)=\cos^3\phi+\cos^3(\phi+2\pi/3)+\cos^3(\phi-2\pi/3)$$
$$-3\cos\phi\cdot\cos(\phi+2\pi/3)\cdot\cos(\phi-2\pi/3)$$

and it vanishes in view of the algebraic identity

$$a^3+b^3+c^3-3abc\equiv(a+b+c)(a^2+b^2+c^2-bc-ca-ab)$$

and the trigonometric identity

$$\cos\phi+\cos(\phi+2\pi/3)+\cos(\phi-2\pi/3)=0$$

which can be proved by direct calculation.

We can even complicate a little the equations of motion (36), admitting a gauging where the velocity $\dot{\boldsymbol{m}}$ has also a component of speed along the ray. Such a component would correspond to a propagation. This is in the spirit of a unitary description of the light phenomenon, which would thus include both the propagation – measured always along the ray – as well as the light motion proper – measured orthogonally to the ray – in describing the light. Mention should be made that such a situation corresponds to the motion of an electric charge in the field of a magnetic pole (Poincaré, 1896). Then the equations of motion corresponding to those from (36) are:

$$\frac{dm_1}{lm_1+p(m_2-m_3)}=\frac{dm_2}{lm_2+p(m_3-m_1)}=\frac{dm_3}{lm_3+p(m_1-m_2)} \tag{46}$$

where $l$ and $p$ are two parameters representing the ‘amounts’ in which the motion is decomposed *along the ray* and *perpendicular* to it, respectively. The integration procedure described above, leads to a differential form a little more complicated than (38), *viz.*:

$$\frac{adm_1 + bdm_2 + cdm_3}{[la + p(c-b)]m_1 + [lb + p(a-c)]m_2 + [lc + p(b-a)]m_3} = \nu dt$$

which can be considered an exact differential:

$$\frac{adm_1 + bdm_2 + cdm_3}{n(am_1 + bm_2 + cm_3)} = \nu dt \quad \therefore \quad am_1 + bm_2 + cm_3 = Ae^{n\nu t} \tag{47}$$

if, and only if, *a*, *b*, *c*are solution of the linear algebraic system given by:

$$(l-n)a + p(c-b) = 0$$

and its positive permutations. This system has nontrivial solutions only if the constants *l*, *p* and *n* satisfy the algebraic equation:

$$(l-n)[(l-n)^2 + 3p^2] = 0$$

which offers three possibilities of construction of the differentials representing the corresponding kinematics. They are given by the system of values:

$$l = n \quad \therefore \quad a = b = c$$

$$l - n = ip\sqrt{3} \quad \therefore \quad a = jc; b = j^2 c$$

$$l - n = -ip\sqrt{3} \quad \therefore \quad a = j^2 c; b = jc$$

Formally, then, nothing changes with respect to the preceding simpler case: it is just that we have here to do with a *harmonic of the frequency ν*, rather than with the frequency itself.

The analogy with the classical case can still be taken to work in this case, because there is a ‘hidden’ dynamics involved here, and this is, we think, the right place to bring about the name of Paul Appell, mentioned above in connection with the Eq. (40). This dynamics appeared for the first time in 1893, in the known *Traité de Mécanique Rationnelle* of Appell, Tome I, on page 351, but only as an exercise. Quoting:

A point is moving in space, under the action of a force whose components *X*, *Y*, *Z* are functions of *x*, *y*, *z*, which verify the relations

$$\frac{\partial X}{\partial x} = \frac{\partial Y}{\partial y} = \frac{\partial Z}{\partial z}, \quad \frac{\partial X}{\partial z} = \frac{\partial Y}{\partial x} = \frac{\partial Z}{\partial y}, \quad \frac{\partial X}{\partial y} = \frac{\partial Y}{\partial z} = \frac{\partial Z}{\partial x} \tag{48}$$

Prove that the integration of the equations of motion is reduced to quadratures [(Appell, 1893), *Exercise 16, p. 351, our translation*].

The proof is simple: first, one has to define a *complex position vector*, having as components the three complex coordinates as in Eq. (40). Then we need to define a *complex force vector*, having as components three corresponding complex quantities, constructed from the real components of force in the same manner the coordinates are constructed. Obviously, the principles of analysis allow us to infer that, if the real forces are functions of real position, the complex forces must be functions of complex positions. Therefore, using our notations for the coordinates along the ray, in the following table constructed by the rules just mentioned:

$$\begin{gathered} x^1 = m_1 + m_2 + m_3, \quad x^2 = m_1 + jm_2 + j^2 m_3, \quad x^3 = m_1 + j^2 m_2 + jm_3 \\ X^1 = X + Y + Z, \quad X^2 = X + jY + j^2 Z, \quad X^3 = X + j^2 Y + jZ \end{gathered} \tag{49}$$

every variable of the second line should be a function of the variables from the first line. Then notice that, under the conditions (48), each of the components of complex force thus defined – assumed conservative, of course – is a function only of the corresponding complex coordinate from the first line. Therefore, the differential equations of motion can be written as

$$\frac{d^2x^1}{dt^2} = X^1(x^1), \quad \frac{d^2x^2}{dt^2} = X^2(x^2), \quad \frac{d^2x^3}{dt^2} = X^3(x^3) \tag{50}$$

and can be solved by integrating twice, indeed. The property is transmitted as such over to the real corresponding quantities, because the transformations (59) are always nonsingular. So the Appell's result is proved.

## 9. A Special Optical Signal

In (Mazilu, 2019), we reproduced an excerpt from Ehrenberg's and Siday's work (Ehrenberg and Siday, 1949), containing a definition of the Aharonov-Bohm effect *avant la lettre*, as we called it there. In fact, taking it a little out of its very words, that excerpt contains a general definition of the action at distance, in the form: *the presence of a charge or a mass, but not the charge or the mass itself, is liable to arise physical phenomena*. This idea is so overrated in physics, that sometimes one can witness *gedanken* experiments whereby an empty universe is invented, with a single particle in it. But the truth is that this is just about the essence of action at a distance as described in physics today: a charge or a mass, present anywhere in the universe, raises phenomena in space anywhere else, and is aroused by such phenomena. This is how physics works, anyway, and the last part of this statement – an induction phenomenon, as it were – is what interest us most here, as part of a phenomenology.

The practice of EEG suggests what we think is a long due analogy between the brain activity and Earth activity. To wit, the Earth's crust is simply analogous to a skull, perhaps even with respect to their electrical properties. For, in view of the practice of EEG, there should be no doubt regarding the fact that the skull has definite electrical properties. The correspondent of these properties in the case of Earth's crust is a little obscure, but, again no doubt, so it is the very physics of the skull. However, what interests us most here, is a certain philosophy of handling a seismogram, that reminds us of a corresponding philosophy of handling an EEG signal. Further on, the brain electric activity as reflected in the EEG correlations between the different locations on the skull (Pribram, 1998) reminds us of those seismographic correlation used in locating the epicenter of the earthquakes in the case of Earth. Thus, in our opinion, it becomes a necessary errand from our part, to try and extend, along this path, the analogy between mechanics and electrodynamics, beyond its usual limits that led to the classical Maxwellian electrodynamics. A few theoretical facts, both geophysical and electrical in nature, to be presented right away, allow us to describe such a possibility of extension with an encouraging impact on the theoretical physics of brain itself.

The relevant mathematical problem amounts down here to the *construction of a correlation between positions along different rays from the space occupied by matter, at different times*. Classically speaking, this is why we need a reference frame, in order to locate a position, and a clock in order to mark the moments of time when we locate that position. Specifically, the reference frame is usually Euclidean, and *the representation of the coordinates as lengths along any three reciprocally orthogonal directions* is just implicit, so to speak. However, while the correlation in general is thought as a kind of controlled exchange between places, here it must be constructed based upon an equation of propagation, which is actually considered as the infinitesimal form of that correlation. In classical electrodynamics as well as in the continuum mechanics of isotropic and homogeneous media, the equation of propagation is the D'Alembert equation. In view of the results of Louis de Broglie presented earlier in this work, we limit our discussion to this equation of propagation.

It is in these circumstances that G.L. Shpilker took notice of the fact that a certain treatment of a real seismogram encompasses a set of procedures which, in our opinion, should be universal from theoretical point of view (Shpilker, 1984). He takes note of the fact that a recorded seismographic signal complicates the act of interpretation beyond the harmonic oscillator structure. Then a conclusion comes out, worth considering in general: *the position of the point of recording must be defined not by lengths, but by three numbers having certain algebraical properties, necessary in order to comply with the definition of the recorded signal*! Taken as the components of a position vector, defined in the sense of Bartolomé Coll, these three numbers actually define a class of phenomena that may be related as well to an electrical activity, with electricity defined as we presented it above. The theory can of course, be applied as such to the physics of brain, and this is quite an important conclusion. Let us see what is this all about.

Shpilker's theory starts, as we said, from the observation that a seismogram can never be represented just by the simple harmonic oscillation which, in the geometry of the D'Alembert's equation, would locally represent a plane wave. As we have shown before, the whole physics associated with an arbitrary time record is contained in the parameters of that harmonic oscillation. A general form of the recorded signal, having any realistic appearance at all, would be as a complex-valued function of a locally devised time sequence that serves to ordering the elongations recorded in the seismogram, like, for instance in Eq. (3.4) of (Mazilu, 2019). In the particular case of linear phase – $\gamma = 0$ in Eq. (3.4) – and no time dependence of the amplitude of signal – $a = 0$ in Eq. (3.4) – such a realistic appearance would be given through a function of time having, in Shpilker's notations, the form:

$$v_1 = \left|A\right| e^{i\alpha_0} e^{(i\omega+\beta)(t-t_0)} \tag{51}$$

with $A$, $t_0$, $\alpha_0$, $\beta$ and $\omega$ – five real constants. They are to be extracted from the data of the seismogram itself. This is, in Shpilker's view, the most general model form of the recording. By comparison with Eq. (3.4) of (Mazilu, 2019), it is quite a particular model, but this is not the point here: it is the *general philosophy of handling this signal*, as developed by Shpilker, that should concern us. It starts from the observation that *the form of this signal as a function of time* is *all we are able to know* positively with a certain degree of confidence: the rest, starting from the very idea of propagation, the equation representing it *etc*., is just a series of hypotheses, educated guesses, is true, but, still, assumptions. However, we need to emphasize it once again from the very beginning, and also certify subsequently via a genuine development, that it is the natural philosophy beyond this procedure which is universal, and should interest us. Let us expound a little more on this statement.

First, the equation of D'Alembert, – or any other equation of propagation for that matter – incorporates only a part of these hypotheses. Admiting, as Louis de Broglie did, that the propagation is described by such an equation, the problem of correlating two points in space is usually solved in physics by the corresponding Green function associated with this equation. However, within Shpilker's philosophy, *i.e.* from the point of view of professional needs we might say, the emphasis is significantly changed: *it falls upon the correlation of the recorded signal with the equation of propagation*, which is a step of knowledge generally bypassed in the regular usage of an equation of propagation. For, it is quite clear that the Eq. (51), which is to be taken as a product of experiment, bestows a *physical content upon the space position where the signal is recorded*. And this physical content is described by Eq. (51) through the intermediary of a local time sequence, in exactly the same manner in which a uniform motion of a classical material point, for instance, describes the physical content of a certain time sequence, obtained locally with an arbitrary clock. Let us analyze the way in which, according to G. L. Shpilker, such a physical content should be brought to bear on the geometry.

Once we have at our disposal the Eqs. (2.1) of (Mazilu, 2019), and (51) here – in general, as we said, an *equation of propagation* and a *physical content of a local sequence of time* – the Shpilker's argument follows quite a simple logic, customary we might say: one must accept that any signal, even a recorded signal, is a solution of the D'Alembert equation, for *this equation defines the concept of signal within matter*, and the recorded signal itself is, obviously, such a signal. The only condition is that the *surface of earthquakes should be a matter surface*, a quality that makes out of it a surface of separation of the matter from space. Notice now that the representation (51) of the recorded signal *does not contain explicitly any space position*, be it that of the source of the seism or of the position of seismograph, but just some parameters to be read out of the recordings. It would be therefore virtually impossible to set this physical content of the signal in connection with the equation of propagation, if one does not assume that the position of the recording point in space is somehow *contained implicitly* among the parameters representing this physical content: $\alpha_0$, $\omega$, $\beta$. Shpilker writes the solution of the D'Alembert equation D in the form:

$$u(\boldsymbol{x},t) = Ae^{\langle \xi | x-y\rangle + c(t-t_0)\sqrt{\langle \xi | \xi \rangle}}; \quad A \equiv \left|A\right| e^{i\alpha_0} \tag{52}$$

where $\boldsymbol{\xi}$ is an arbitrary complex vector. Obviously, this solution satisfies D'Alembert equation, both in the variables $(\boldsymbol{x},t)$, and in the variables $(\boldsymbol{y},t_0)$, *no matter of the vector* $\boldsymbol{\xi}$ *and amplitude A*, which are here taken as complex quantities. Consequently, the Eq. (52) actually *represents a correlator analogous to the classical Green function*, of two 'legal signals', whose 'legality' is defined here according to a precise criterion though: every *admissible signal* must be a solution of D'Alembert's equation. From theoretical point of view this signal must be found all over the places within matter, therefore both at the location of emission and the location of recording. Except that now the functional form of the signal at the emission position, or in fact during propagation, is somewhat more realistic, inasmuch as it is not *a priori* defined – for instance by analogy with a mechanical model, as we did in Chapter 3 of (Mazilu, 2019) – but empirically, with a physical content defined *in the manner we define the recorded signal*.

Now the solution of our problem comes down to matching the theoretical representation (52) with the recorded signal from Eq. (51). In order to do this, Shpilker uses the freedom offered by the arbitrariness of the vector $\boldsymbol{\xi}$: in the surface delimiting the Earth seismically – the *surface of quakes*, as we would like to call it – he takes the signal as being of the form

$$v(\boldsymbol{x},t) = Ae^{\langle z\otimes(k+il) | x-y\rangle + (i\omega+\beta)(t-t_0)} \tag{53}$$

This signal reduces to that from Eq. (51) for $|x-y\rangle = |0\rangle$, which means that $|y\rangle$ *may be taken as the position of the point of recording*. Then, again, the function (53) should be a solution of the D'Alembert equation. This time, however, special conditions must be secured, whereby over the recorded signal one overlays another signal, which needs to be conveniently described in order to account for the conditions in which the measurement is performed.

Before any explanation on these conditions, a word about the notations from Eq. (53): the vector $\boldsymbol{z}$, as well as $\boldsymbol{\xi}$ for that matter, is unknown. The vector $\boldsymbol{k}+i\boldsymbol{l}$ is an arbitrary complex vector, submitted by Shpilker to the constraints:

$$k_1^2 + l_1^2 = k_2^2 + l_2^2 = k_3^2 + l_3^2 = \tau^2 \tag{54}$$

where $\tau$ is an arbitrary real number. Further on, one denotes

$$|z \otimes (k + il)\rangle \equiv \begin{pmatrix} z_1(k_1 + il_1) \\ z_2(k_2 + il_2) \\ z_3(k_3 + il_3) \end{pmatrix} \tag{55}$$

so that this is just *a complex vector with its components given by the diagonal entries of the complex matrix* $\boldsymbol{z}\otimes(\boldsymbol{k}+i\boldsymbol{l}) \equiv |z\rangle\langle k+il|$.

Now, coming back on the track of our discussion, Shpilker claims that in order to get a correct 'reconstruction' of the field from the recorded signal, as defined by Eq. (51), the coordinates of the position of recording must be expressed by the ratios:

$$y_j = l_j / k_j; \quad j = 1,2,3 \tag{56}$$

submitted to the conditions

$$y_2 \neq y_3, \quad y_3 \neq y_1, \quad y_1 \neq y_2 \tag{57}$$

which are thus *necessary* and *sufficient* for a reconstruction of the field from recorded data. Therefore Shpilker's local coordinates *are not regular coordinates*. It is more proper to say that *they define in fact a reference frame*: that reference frame in which the coordinates of the position of the recording point with respect to the point where the signal is originating, are given by the vector $\boldsymbol{y}$ having the components (56), submitted to the conditions from Eq. (57). The question is, what are these coordinates, from a physical point of view, and this depends entirely on the physical quantity we are measuring locally. This is the whole point of Shpilker's natural philosophy: *the location is pinpointed in space by the very quantity to be perceived at that location*!

Before anything else along this line, let us finish the job we started here: to show that in order to have a solution of the problem in the form from Eq. (53), the Shpilker's demands have to be met, indeed. In order to prove this, notice that from the Eqs. (51) and (52) one gets:

$$u(\boldsymbol{y},t) = v_1 \quad \therefore \quad \begin{cases} c^2(\xi_1^2 + \xi_2^2 + \xi_3^2) = \beta^2 - \omega^2 + 2i\beta\omega \\ c^2(|\xi_1|^2 + |\xi_2|^2 + |\xi_3|^2) = \beta^2 + \omega^2 \end{cases} \tag{58}$$

by the virtue of the fact that $\boldsymbol{\xi}$ has complex components in general. In fact, according to Eq. (53) these components are defined by Eq. (55), so that the arbitrariness of the vector $\boldsymbol{\xi}$, having six real components, is transferred into the ambiguity of the vectors $\boldsymbol{z}$, $\boldsymbol{k}$ and $\boldsymbol{l}$, which involve nine real components. The situation becomes normal if we have three relations connecting these last two vectors, which must be *measurement constraints*, as those given by Shpilker in Eq. (54), which can be taken just naturally as such. Therefore, using the Eqs.

(56) in (58), the components of vector $z$ can be found as solutions of the linear system: which is obviously equivalent to the system:

$$\begin{aligned} & c^2(k_1^2 z_1^2 + k_2^2 z_2^2 + k_3^2 z_3^2) = \beta^2 \\ & c^2(k_1 l_1 z_1^2 + k_2 l_2 z_2^2 + k_3 l_3 z_3^2) = \beta\omega \\ & c^2(l_1^2 z_1^2 + l_2^2 z_2^2 + l_3^2 z_3^2) = \omega^2 \end{aligned} \tag{59}$$

This system is in turn compatible, and has unique solution if, and only if, its principal matrix is nonsingular. The determinant of this matrix can be easily calculated, and gives:

$$\begin{vmatrix} k_1^2 & k_2^2 & k_3^2 \\ k_1 l_1 & k_2 l_2 & k_3 l_3 \\ l_1^2 & l_2^2 & l_3^2 \end{vmatrix} \equiv (k_2 l_3 - k_3 l_2)(k_3 l_1 - k_1 l_3)(k_1 l_2 - k_2 l_1) \tag{60}$$

Therefore the compatibility of the system (59) comes down to the fact that none of the projections of the real vector $\boldsymbol{k}$ on the planes of coordinate should be collinear with the corresponding projection of the real vector $\boldsymbol{l}$. Solving the system (59) results in

$$z_1^2 = \frac{1}{c^2} \frac{l_2 l_3 \beta^2 - (k_2 l_3 + k_3 l_2)\beta\omega + k_2 k_3 \omega^2}{(k_3 l_1 - k_1 l_3)(k_1 l_2 - k_2 l_1)}$$

and its even permutations over indices 1, 2, 3. Considering now the definitions from Eq. (56), we will have right away:

$$z_1^2 = \frac{1}{c^2 k_1^2} \frac{y_2 y_3 \beta^2 - (y_2 + y_3)\beta\omega + \omega^2}{(y_3 - y_1)(y_1 - y_2)} \tag{61}$$

and two more relations, given by the even permutations over indices 1,2,3. This proves the necessity and sufficiency of the Shpilker's conditions (57), showing, moreover, that the vector $\boldsymbol{k}$ must have all its components nonnull for a reconstruction of the field in finite terms. Consequently, the triple $|y\rangle$ represents here the position of recording point of the earthquake, according to its definition contained in the particular relation between D'Alembert equation – describing the signal propagation – and the functional form of the recorded signal.

For G. L. Shpilker – as well as for the whole geophysicists' community, in fact – such a solution of the problem of quakes is essential. Indeed, the seismogram is actually a *single* closed-form expresssion of a limiting condition in space and time for an equation of propagation – the case in point is D'Alembert equation – and, for instance, a conceivable Cauchy problem of this case cannot be solved. Such a solution needs boundary conditions in multiple points on a surface. Usually, for solving the problem for the case in point, one would need conditions over *the entire surface of the Earth,* defined by the existence of quakes. First of all, such a surface cannot be defined itself, even if we disregard the idea of seismogram, to say nothing of the fact that one cannot place seismographs all over the places where an earthquake is felt, in order to make the necessary measurements. It is therefore instrumental, indeed, to built a signal as the solution of the equation of propagation, *starting from data recorded sporadically*, insular data at best. Which is what Shpilker's theory accomplishes in a brilliant way. This approach has, however, much more general connotations, even fundamental we should say, from the point of view of the theoretical physics. These connotations can be extracted even limiting ourselves to the classical differential geometrical idea of adaptation of a reference frame to a surface embedded in space (Delphenich, 2013a; Delphenich, 2013b).

There is not too much to add over what we just said above, in order to see in this approach of handling the seismic signal, an opportunity of extension to the brain EEG case. The coordinates $\boldsymbol{y}$, in particular, can be useful in the theory of *evoked potentials*. In seismology they give the coordinates of recording point with respect to the point of initiation of the quake. In the case of EEG, they might as well represent the coordinates of the point of initiation of the neuronal activity having as result the skull recording. Only, in this last case we need to be assured that the vector $\boldsymbol{y}$ is somehow related to an electric or magnetic activity, which is indeed the general case, if we connect its components with the split angle of the charge induced by a recorded signal in a place on the skull.

In the previous development we have followed as much as possible the Shpilker's notations. However, even in these notations, it is not too much to say over what has already been said, in order to see that in Eq. (54) the parameter $\tau$ can be identified with the charge (31), and the three Shpilker coordinates (56) can be, in fact, three arbitrary charge splits of the very same charge. This interpretation corresponds to a philosophy of the EEG, quite natural we should say: the electric signal, initiated somewhere in the brain is perceived on the skull, just like the seismic signal. It creates a local charge, and this is actually measured. Any three splits of this charge define, as in Eq. (35), three coordinates of the recording point with respect to the point of initiation of the signal. The coordinates of such a location are by no means unique, but this arbitrariness can be tied up with the arbitrariness of the reference frame with respect to which we locate the position on the skull.

In order to carry on physically and mathematically such a natural philosophy, a little inventory of the minimal necessities may be in order. First, in the order of things necessary, comes the *definition of the general signal* to replace that from Eq. (51). If the mechanical analogy is to be maintained – and everything recommends it – then the signal given in Eq. (3.4) of (Mazilu, 2019) is our best candidate, and we shall pursue this idea, under the guidance of Shpilker example, of course. Second in the order of things necessary, comes an *appropriate equation of propagation*, in order to replace, more realistically, the idealistic D'Alembert equation. Just as in the case of Schrödinger equation for the case of interpretation, our best candidate is the *heat equation*, inasmuch as it satisfies to the same invariance group as the Schrödinger equation. Besides, its theoretical handling asks for a definite relationship between space and time measures, which is instrumental for a physical theory. Thirdly, the Shpilker's natural philosophy specifically asks for solving an issue raised by the practice of EEG: the connection between the *coordinate along the ray* and *any other geometrical coordinate* we may conveniently use in the physics of brain. A gauging procedure defining static electromagnetic fields of the nature of distributed charges, may add to our phenomenological understanding of the problems.

## 10. A Yang-Mills Gauging

Zenaida Uy's gauging procedure (Uy, 1976) can be taken as a procedure of defining some electromagnetic quantities, indeed, related to the Shpilker coordinates of the point of recording. In view of the fact that in EEG the recording point is on the skull, the electromagnetic quantities in question should be thus connected with the electric properties of the skull itself. But there is more to it: as it turns out, the very same coordinates must be also connected with some magnetic properties. And thus, they can be made responsible for the shape of the magnetic signals recorded in a EMG around the skull.

In order to show this, let us describe the Uy's scaling procedure. It is connected with a geometry of the quadratic quantities ($y_2y_3$, $y_3y_1$, $y_1y_2$) which appear in the definition (61) above or, in fact in the Eqs. (41) and (42) in connection with Appell coordinates. To start with, if one identifies Shpilker coordinates with the components of the vector $\boldsymbol{m}$, the Eqs. (36) can be written in the compact form

$$\dot{y}_k + v\langle 1|\boldsymbol{h}_k|y\rangle = 0; \quad \langle 1| \equiv (1,1,1) \tag{62}$$

Here the 3×3 matrices $\boldsymbol{h}_k$ are constructed from the structure constants of the rotation group according to the rule: $(\boldsymbol{h}_k)_{ij} = \varepsilon_{kij}$, where the totally skew-symmetric Levi-Civita tensor $\boldsymbol{\varepsilon}$. This definition results in the following three matrices making a closed algebraic system, indeed, which generates an exponential part of the three-dimensional rotation group in the Euclidean space, the part connected with the identity matrix:

$$\boldsymbol{h}_1 \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}; \boldsymbol{h}_2 \equiv \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \boldsymbol{h}_3 \equiv \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{63}$$

These matrices are, indeed, a three-dimensional linear basis in the space of skew-symmetric 3×3 matrices. Choose now the vector having components of second degree mentioned above, as the column matrix:

$$\boldsymbol{b}_0 \equiv \begin{pmatrix} y_2 y_3 \\ y_3 y_1 \\ y_1 y_2 \end{pmatrix} \tag{64}$$

The action of the basis matrices (63) on this vector defines three new vectors

$$\boldsymbol{h}_1 \cdot \boldsymbol{b}_0 \equiv y_1 \begin{pmatrix} 0 \\ y_2 \\ -y_3 \end{pmatrix}; \boldsymbol{h}_2 \cdot \boldsymbol{b}_0 \equiv y_2 \begin{pmatrix} -y_1 \\ 0 \\ y_3 \end{pmatrix}; \boldsymbol{h}_3 \cdot \boldsymbol{b}_0 \equiv y_3 \begin{pmatrix} y_1 \\ -y_2 \\ 0 \end{pmatrix} \tag{65}$$

Zenaida Uy builds then the following *four* vectors

$$\begin{gathered} \boldsymbol{b}_1 \equiv f(\boldsymbol{y}) \cdot (\boldsymbol{h}_1 \cdot \boldsymbol{b}_0); \quad \boldsymbol{b}_2 \equiv f(\boldsymbol{y}) \cdot (\boldsymbol{h}_2 \cdot \boldsymbol{b}_0); \quad \boldsymbol{b}_3 \equiv f(\boldsymbol{y}) \cdot (\boldsymbol{h}_3 \cdot \boldsymbol{b}_0) \\ \boldsymbol{b}_4 \equiv g(\boldsymbol{y}) \cdot (\boldsymbol{1} \cdot \boldsymbol{b}_0) \end{gathered} \tag{66}$$

where $f$ and $g$ are two functions, arbitrary for the moment. Then she takes notice of the important fact that *the tetrad* ($\boldsymbol{b}_1$, $\boldsymbol{b}_2$, $\boldsymbol{b}_3$, $\boldsymbol{b}_4$) can be so chosen as to represent *static Yang-Mills* SU(2) fields. As usually acknowledged, the Yang-Mills fields, in general, are to be considered as the natural generalization of the classical electromagnetic fields, to which one adds the properties brought about by the relativistic and wave-mechanical concepts. However we take them here as they were considered at the moment they were discovered: it is this take that can have any meaning for the physics of brain, as we shall see in the present work. Quoting the abstract of the original work of Chen-Ning Yang and Robert Mills:

> It is pointed out that the usual principle of invariance under isotopic spin rotation *is not consistent with the concept of localized fields*. The possibility is explored of having *invariance under local isotopic spin rotations*. This leads to formulating *a principle of isotopic gauge invariance* and the existence of a ***b*** field which *has the same relation to the isotopic spin* that *the electromagnetic field has to the electric charge*. The ***b*** field satisfies nonlinear differential equations. The quanta of the ***b*** field are *particles with spin unity*, and *electric charge* ±*e* or zero (Yang & Mills, 1954).

It is best to explain *our incentives* in considering the Yang-Mills fields here, in connection with this agenda of the renowned work just cited.

For the moment being it is sufficient to declare that these incentives originate all from the idea of memory as defined holographically. A hologram represents a 'deposit' as it were, of a space figure on a certain surface. However, rarely, if ever, is it noticed that this 'deposit' shares with the space original its dimensionality. Specifically, it is three-dimensional just like the space figure it describes, and this fact can be revealed algebraically [(Mazilu *et al.*, 2019), Chapter 7]. The physical properties are quite specifically deposited on a surface: *in the local variation of its curvature parameters*. Now, the matrices describing the physical variations of curvature, share with the isotopic classical fields the property of three-dimensionality, and this shall be further detailed in the present work. The physics itself can be connected to the space and time only via a reference frame, and this is the whole morale of Zenaida Uy's gauging procedure. In short, it solves one of the important issues of the definition of a reference frame: it has to be defined by a statical condition, to which any dynamics should be referred.

The gauge field intensities related to $\boldsymbol{b}$ fields (66) are given by a generalization of the classical electrodynamics relations. These are modified according to Yang-Mills nonabelian prescription (Yang and Mills, 1954); see also (Wu and Yang, 1969):

$$\boldsymbol{f}_{\mu\nu} \equiv \partial_\mu \boldsymbol{b}_\nu - \partial_\nu \boldsymbol{b}_\mu - \boldsymbol{b}_\mu \times \boldsymbol{b}_\nu \tag{67}$$

under the following 'equations of motion', as they are usually called:

$$\partial_\nu \boldsymbol{f}_{\mu\nu} + \boldsymbol{b}_\nu \times \boldsymbol{f}_{\mu\nu} = 0; \quad \partial_\nu \boldsymbol{b}_\nu = 0 \tag{68}$$

Here, the usual summation over repeated indices is assumed. The static feature of the field is explicitly recognized in the fact that the tetrad $|\boldsymbol{b}\rangle$, and the corresponding field intensities (67), do not depend explicitly on any 'time' coordinate, $y_4$ say, that might incidentally complete the position $\boldsymbol{y}$ to an event. Zenaida Uy specifically assumes that the functions $f$ and $g$ depend on coordinates via the 'volume' $\omega = y_1 \cdot y_2 \cdot y_3$ of the cuboid whose diagonal is the vector $\boldsymbol{y}$, and, additionally, that $g = \pm f$. One can then calculate the 'electric' and 'magnetic' gauge field intensities associated to the tetrad $|b\rangle$, by the following prescriptions, which replicate the well-known classical definition of electromagnetic field intensities (Wu & Yang, 1969):

$$\boldsymbol{E}_k \equiv i\boldsymbol{f}_{k4}; \quad \boldsymbol{H}_k \equiv \frac{1}{2}\varepsilon_{kij}\boldsymbol{f}_{ij} \tag{69}$$

Calculating effectively the electric field here, with Eq. (66) and the definition (67) of field intensities, we have the result:

$$\boldsymbol{f}_{k4} = -\partial_k \boldsymbol{b}_4 - \boldsymbol{b}_k \times \boldsymbol{b}_4 \quad \therefore \quad \boldsymbol{E}_k = -i\left[(\Sigma y_2 y_3)(g^{-1}g')\boldsymbol{b}_4 + g\boldsymbol{e}_k\right] \tag{70}$$

where the summation runs over the positive permutations of the numerical indices, and a prime means derivative with respect to the unique variable – in this case $\omega$ – as usual. It is quite important to take notice here of the form of the vectors that we denoted in Eq. (70) by $\boldsymbol{e}_k$. These are given by the columns of a quasi-orthogonal matrix that plays a crucial part in the *space inversions*. Such a matrix is, indeed, of the form involved in the Maxwell's construction of the electromagnetic stresses, and we called 'equivalent to a vector' [see (Mazilu *et al*., 2019), Eqs. (5.22) ff.]:

$$\boldsymbol{e}_{ij} = y_i y_j - \frac{1}{2}\boldsymbol{y}^2\delta_{ij} \tag{71}$$

We shall return later to this important issue. For now, let us calculate the magnetic fields: using again (66), (67), and the second of (69), we further have

$$\boldsymbol{f}_{ij} = \omega f^2(\omega)\varepsilon_{ijk}y_k \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \quad \therefore \quad \boldsymbol{H}_k = \omega f^2(\omega) y_k \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \tag{72}$$

Here we have to notice that the second condition from Eq. (68) is an identity, for we have

$$\partial_\nu \boldsymbol{b}_\nu \equiv \partial_k \boldsymbol{b}_k = [\omega f'(\omega) + f](\Sigma \boldsymbol{u}_k)$$

where $\boldsymbol{u}_k$ are the column matrices from Eq. (5.35). Now, in view of the obvious fact that $\Sigma \boldsymbol{u}_k = \boldsymbol{0}$ the result is automatically the null vector.

The *geometric* interpretation of this situation is as follows. Take the elementary vector surface 1-forms:

$$ds^k = \varepsilon^{klm} y_l dy_m \tag{73}$$

They can be written as the bilinear forms, using the ***h***-matrices from Eq. (63):

$$ds^k \equiv \langle y | \boldsymbol{h}^k | dy \rangle \tag{74}$$

so that the column matrices used in the definitions of the vectors from (65) are, in fact, defined by identities $\boldsymbol{u}_k \equiv \boldsymbol{h}_k \cdot |y\rangle$. The problem now, is the presence in our theory of the vector from Eq. (64). As one can see, however, a justification is not altogether out of hand: it is the essential vector for the absolute geometry based upon classical idea of volume. For instance, we get the important gauging relation mentioned above, by noticing the dot product

$$\boldsymbol{y} \cdot \boldsymbol{b}_0 = 3y_1 y_2 y_3 \tag{75}$$

which allows for a statistical discussion of electromagnetic stresses as fluxes.

## 11. Conclusions

The living brain should be physically modelled as a universe, analogous with the physical universe of today's cosmology, and we describe the analogy in its essentials. The analogy is mathematically based on the idea of interpretation of the universe, the only positive definition of which is provided by the wave mechanics (see I for details on the concept of interpretation). According to this idea, a universe can be interpreted as a homogeneous ensemble of free classical particles. As a static ensemble, such an ensemble can be thought in terms of identical classical particles endowed with gravitational mass and two charges, electric and magnetic. Then between these particles there are Newtonian forces in equilibrium in any direction at any distance, provided a certain quadratic expression involving the physical characteristics of particles vanishes. We call this quadratic expression *inertial mass*. A universe, in general, can be mathematically described according to the sign of inertial mass of the particles of its interpretative ensemble. A positive inertial mass means an actual physical structure of the universe, whereby the Newtonian forces are dominating at any space and time scale. If under this condition, if the gravitational forces prevail, then we have the usual physical universe of cosmology. On the other hand, if for positive inertial mass, the electric forces prevail, we have to do with the universe of brain.

The analogy goes deeper, thus making possible a proper physical theory of the brain. First, the constitutive unit of brain matter – the neuron – handles the charges within brain just as the rays of light in the regular physical optics handle the Bartolomé Coll's luxons in the cosmological universe of physics (see I). However, inasmuch as the charges are electric as well as magnetic, phenomenologically corresponding to the two possibilities of non-destructive experimental access to brain, we need to handle the physics of brain accordingly. As it turns out, though, this handling has nothing of the kind the actual theoretical physics does not know. This second part of our work develops, for instance, the mathematics of recorded signals from the brain by analogy with the mathematics of seismic recording in the physics of Earth's crust, which thus can be taken as the analogue of the skull, or *vice versa*, if one likes to. And, sure enough, this mathematics is prone to a general elucidation by Yang-Mills fields, even though defined by a special scaling procedure. As life has it, this scaling procedure is long known in theoretical physics! In hindsight, the special electrodynamics thus built may be able to explain, for instance, the concentration of social life in the places of high seismic activity, a circumstance helping us, later on along the proceedings of the present work, in defining the concept of infinity for the brain model universe.

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## PRINCIPII FIZICE ÎN EVIDENȚIEREA MECANISMELOR DE FUNCȚIONARE A CREIERULUI. PARTEA A II-A

(Rezumat)

În prezenta lucrare creierul este modelat fizic ca un univers, analog modelelor fizice cosmoligice existente. Analogia este explicitată matematic cu ajutorul unei teorii

ce implică mărimi fizice ce descriu materia din punct de vedere fenomenologic. Într-un astfel de context, sarcina, care este proprietatea fundamentală a materiei creierului viu, ar trebui să aibă aceeași origine ca masa gravitațională: este „creată” sub influența materiei. Un astfel de model permite corespondența dintre masa gravitațională, sarcina electrică și masa inerțială. Această masă este controlată prin intermediul materiei aflate la infinit, în acord cu principiul general al lui Mach. În consecință, această analogie ne permite să definim semnalele EEG și MEG, utilizate pentru dinamicile cerebrale. Aceste semnale sunt definite la fel ca semnalele seismice: scoarța terestră prin urmare poate fi asociată structurilor craniene. Mai mult, prezentăm un câmp Yang-Mills staționar ca o instanță a unui câmp electromagnetic al acestui univers.